

# NC 227A: INTEGRATED MATH IIA

## Citrus College Course Outline of Record

Heading	Value
Effective Term:	Fall 2022
Credits:	0
Total Contact Hours:	60
Lecture Hours :	60
Lab Hours:	0
Hours Arranged:	0
Outside of Class Hours:	120
Prerequisite:	Placement by high school counselor or math placement exam.
Transferable to CSU:	No
Transferable to UC:	No
Grading Method:	Non-Credit Course

## Catalog Course Description

The focus of the Mathematics II course is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, Statistics and Probability. Students will be focusing on five key elements: (1) extend the laws of exponents to rational exponents; (2) compare key characteristics of quadratic functions with those of linear and exponential functions; (3) create and solve equations and inequalities involving linear, exponential, and quadratic expressions; (4) extend work with probability; and (5) establish criteria for similarity of triangles based on dilations and proportional reasoning. 60 lecture hours.

## Course Objectives

- Make sense of problems and persevere in solving them.
- Analyze givens, constraints, relationships, and goals.
- Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- Make sense of quantities and their relationships in problem situations.
- Bring two complementary abilities to bear on problems involving quantitative relationships.
- Create a coherent representation of the problem at hand.
- Construct viable arguments and critique the reasoning of others.
- Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed.
- Analyze situations by breaking them into cases. Recognize and use counterexamples using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- Recognize the significance of an existing line in a geometric figure and use the strategy of drawing an auxiliary line for solving problems.
- Evaluate the reasonableness of their intermediate results.

## Major Course Content

1. Algebra
  - a. See the structure of expressions.
2. Interpret the structure of expressions.
  - a. Interpret expressions that represent a quantity in terms of its context.
    - i. Interpret parts of an expression, such as terms, factors, and coefficients.
    - ii. Interpret complicated expressions by viewing one or more of their parts as a single entity. [For example, interpret  $x^2 + 6x + 9$  as the product of  $P$  and a factor not depending on  $P$ .]
  - b. Use the structure of an expression to identify ways to rewrite it. For example, see  $x^2 - 4x + 4$  as  $(x - 2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x - 2)(x - 2)$ .
  - c. Arithmetic with Polynomials and Rational expressions
3. Perform arithmetic operations on polynomials
  - a. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
  - b. Geometry
    - i. Congruence
4. Prove geometric theorems (formal proofs)
  - a. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
  - b. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
  - c. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
    - i. Similarity, Right Triangles, and Trigonometry
5. Understand similarity in terms of similarity transformations, including dilations
  - a. Verify experimentally the properties of dilations given by a center and a scale factor:
    - i. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    - ii. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
  - b. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- c. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.
6. Prove theorems involving similarity (formal proofs)
  - a. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
  - b. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
7. Define trigonometric ratios and solve problems involving right triangles
  - a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
  - b. Explain and use the relationship between the sine and cosine of complementary angles.
  - c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
8. Derive and use the trigonometric ratios for special right triangles ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$  and  $45^\circ$ ,  $45^\circ$ ,  $90^\circ$ ).
  - a. Circles
9. Understand and apply theorems about circles.
  - a. Prove that all circles are similar.
  - b. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
  - c. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
  - d. Construct a tangent line from a point outside a given circle to the circle.
10. Find arc lengths and areas of sectors of circles.
  - a. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
    - i. Express Geometric Properties with Equations
11. Translate between the geometric description and the equation for a conic section (parabolas and circles only)
  - a. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
  - b. Derive the equation of a parabola given a focus and directrix.
12. Use coordinates to prove simple geometric theorems algebraically.
  - a. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ . [Include simple circle theorems.]
  - b. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
    - i. Geometric Measurement and Dimension
13. Explain volume formulas and use them to solve problems.
  - a. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
  - b. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
  - c. Know that the effect of a scale factor  $k$  greater than zero on length, area, and volume is to multiply each by  $k$ , respectively; determine length, area and volume measures using scale factors.
  - d. Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems.
  - e. Statistics and Probability
    - i. Conditional Probability and the Rules of Probability
14. Understand independence and conditional probability and use them to interpret data (link to data from simulations or experiments)
  - a. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
  - b. Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - c. Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .
  - d. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.)
  - e. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
15. Use the rules of probability to compute probabilities of compound events in a uniform probability model.
  - a. Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.
  - b. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.
  - c. Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.
  - d. Use permutations and combinations to compute probabilities of compound events and solve problems.
    - i. Use Probability to Make Decisions
16. Use probability to evaluate outcomes of decisions (drawing by lots, random number generators, product testing, sports strategies -pulling the goalie at the end of the game)

- a. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- b. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Examples of Required Writing Assignments

When looking at inverse functions, students will be asked to write an explanation on: 1) How they get from the red graph (original) to the blue graph (inverse); 2) How is the line  $y = x$  (green) used to find the inverse? 3) Think back to Integrated 1... , describe in words what the vertical line test tells you about a graph.

## Examples of Outside Assignments

There will be homework given on a daily basis along with some performance tasks that will need additional time outside of class.

## Instruction Type(s)

Lecture, Online Education Lecture